UNIVERSITY OF PORTO - FACULTY OF ENGINEERING

GEOMETRICALLY NON-LINEAR OSCILLATIONS OF COMPOSITE LAMINATED PLATES BY THE HIERARCHICAL FINITE ELEMENT METHOD

by

Rui Pedro Monteiro Amaro Duarte, with a degree in Apllied Mathematics,

University of Coimbra, Faculty of Sciences

Thesis submitted in fulfilment of the degree of

Master

In Computational Methods in Sciences and Engineering

Thesis under the supervision of Dr. Pedro Leal Ribeiro,
From the Department of Mechanical Engineering and Industrial Management, Section of Applied Mechanics from the Faculty of Engineering of University of Porto

Porto, October 2004

SUMMARY

The geometrically non-linear forced vibration of fully clamped composite laminated plates is studied by the hierarchical finite element method (HFEM). Using the first order shear deformation theory (FSDT), Kirchhoff's hypothesis is relaxed by removing the third part, i.e., the transverse normals do not remain perpendicular to the midsurface after deformation. Von Kármán's non-linear strain-displacement relationships are employed and the middle plane in-plane displacements are included in the model, as well as the rotations about the in-plane axis x and y. The equations of motion are developed in the time domain by applying the principle of virtual work. The high order polynomials that emerge in the HFEM are integrated by symbolic manipulation. These equations are solved in the time domain using Newmark direct integration scheme. The time domain response is studied using the phase plane, Poincaré maps, Fourier spectra and Lyapunov exponents; periodic, quasi-periodic and chaotic motions are obtained. Two different types of forces are considered, the results for linear and non-linear analysis are compared with published ones and good agreement is found. It is demonstrated that the HFEM requires fewer degrees of freedom (DOF) than the more common *h*-version of the FEM. This is a very important advantage in non-linear analysis because the time required to solve the non-linear equations of motion increases significantly with the number of DOF.

SOMMAIRE

On s'intéresse ici à l'étude, à l'aide de la Méthode des Eléments Finis Hiérarchiques (MEFH), des vibrations forcées, en non-linéaire géométrique, de plaques composites encastrées. La modélisation utilisée repose sur la théorie de Reissner-Mindlin (déformation au première ordre), avec l'hypothèse de Kirchhoff où le troisième terme a été négligé (les normales au feuillet moyen ne sont plus nécessairement perpendiculaires à ce feuillet après déformation) et en considérant la relation non-linéaire tension-déplacement de Von Kármán. Les déplacements du feuillet moyen ainsi que les rotations suivant les axes x et y sont introduits dans le modèle. Les équations du mouvement sont obtenues par application du principe des travaux virtuels. Les polynômes d'ordres élevés qui interviennent dans la méthode des éléments finis hiérarchiques sont pris en compte par manipulation symbolique. Les équations du mouvement sont résolues numériquement dans le domaine temporel par la méthode de Newmark. Les réponses dans le domaine temporel sont analysées en étudiant le comportement des trajectoires dans le plan de phase, des sections de Poincaré, des spectres de Fourier et par calcul des exponents de Lyapunov. Deux types sollicitations externes sont considérés. Les réponses périodiques, quasi-périodiques et chaotiques sont mises en évidence. Les résultats pour l'analyse linéaire et non-linéaire sont comparés avec ceux publiés et un bon accord est trouvé. Le coût numérique pour résoudre les équations non-linéaires du mouvement augmente considérablement avec le nombre de degrés de liberté. Ceci est un inconvénient pour mener l'analyse de systèmes faiblement amortis où des simulations sur un temps long sont exigées. La méthode des MEFH nécessitant moins de degrés de liberté que la classique MEF version-*h*, elle est fortement indiquée pour l'analyse numérique dans le domaine temporel des oscillations non-linéaires des plaques composites laminées.

SUMÁRIO

A vibração forçada em regime não linear geométrico de placas assimétricas encastradas em materiais compósitos é estudada pelo método dos elementos finitos hierárquico (MEFH). A origem do sistema de eixos encontra-se no plano médio, sendo o eixo dos *zz* normal a este. Usando a teoria de Mindlin para placas (*first order shear deformation theory*), a hipótese de Kirchhoff é relaxada removendo a terceira parte, i.e, os deslocamentos transversos não se mantêm perpendiculares à superfície média, após deformação. As relações não lineares de von Kármán entre as deformações e os deslocamentos são aplicadas e os deslocamentos ao longo do plano médio são incluídos no modelo, bem como as rotações dos deslocamentos transversos ao longo do eixo dos *xx* e dos *yy*. As equações de movimento no domínio do tempo são determinadas aplicando os princípios dos trabalhos virtuais. Os polinómios de ordem superior que aparecem no MEFH são integrados usando manipulação simbólica. Estas equações são resolvidas no domínio do tempo usando o método de Newmark. A estabilidade da solução obtida, no domínio do tempo, é estudada, usando planos de fase, mapas de Poincaré, espectro de Fourier e expoentes de Lyapunov. Soluções periódicas, quase-periódicas e caóticas são obtidas. Dois tipos de forças são consideradas, os resultados obtidos em análise linear e não linear são comparados com outros publicados e boa concordância é encontrada. É demonstrado que o MEFH requer menos graus de liberdade (GL) que a versão-*h* do método. Esta é uma grande vantagem em análise não linear porque o tempo necessário para resolver as equações não lineares de movimento aumenta significativamente com o número de graus de liberdade.

TABLE OF CONTENTS

LIST OF FIGURES

CHAPTER 2

CHAPTER 4

CHAPTER 5

LIST OF TABLES

CHAPTER 4

ACKNOWLEDGEMENTS

I wish to thank:

My supervisor in this thesis, Professor Pedro Leal Ribeiro, for the time spent helping me to enter in the field on non-linear dynamics. His help and encouragement kept me going up to this day.

My mother Clídia Duarte and my father José Duarte and my sister Patrícia Duarte, who gave me love, affection, education and "*the tools to build a life*".

My lovely wife Alexandra, for the love and encouragement she always gave me. Her patience and devotion are something I shall never forget.

The support of Superior School of Engineering of Viseu and the Portuguese Science and Technology Foundation is gratefully acknowledge.

I dedicate this thesis to:

The memory of my grandmothers Júlia Amaro and Isaura Duarte, grandfathers José Amaro and Joaquim Duarte.

My son, Miguel Tomás Duarte, who will become the most important person in my life.