Chapter 1

INTRODUCTION

1. *GENERAL INTRODUCTION*

Composite laminated structures are used in many areas of engineering such as aeronautics, space engineering and naval industry [1.1, 1.2, 1.3]. Those structures are submitted to vibrations with large amplitude, therefore in the geometrically non-linear regime. Nowadays, typical examples of composite plates are the panels used in spaceships, aircrafts and automobiles which are submitted to large acoustic, aerodynamic and inertia excitation [1.4]. Vibrations with large amplitude cause large tensions and the diminution of life due to fatigue. Quasi periodic and chaotic behaviours are other consequences of non-linearity, completely ignored by linear models which are normally used [1.5, 1.6]. Therefore the study of geometrically non-linear vibration in laminated plates made of composite and hybrid materials becomes important.

A hierarchical finite element method developed recently [1.7 - 1.9], in which the model is improved by increasing the number of shape functions in each element, needs a small number of degrees of freedom. This is a big advantage, because the non-linear equations of motion are solved using iterative methods.

2. *REVIEW OF RESEARCH CARRIED OUT ON PLATE VIBRATIONS*

The combination of two or more materials in order to form a new material with better properties is something that can be understood as a simple definition of composite materials. There are various types of composite materials: fibre, particulate, laminar, flake and filled [1.10]. Laminated composite materials are made of layers with different materials, which include fibrous composites and particulate materials. In this case, composites can be either metallic or non-metallic. Thus there are four possible combinations: metallic in non-metallic, non-metallic in metallic, non-metallic in non-metallic and metallic in metallic.

A lamina is a sheet of composite material. A fibre reinforced lamina consists of many fibres embedded in a matrix material like aluminium, or non-metal like a thermoset or thermoplastic polymer. The fibres can be continuous or discontinuous, wover, unidirectional, bidirectional or randomly distributed. Unidirectional fibre reinforced lamina exhibit the highest strength and stiffness in the direction of the fibres, but in the transverse direction of the fibres they have very low strength and stiffness. Poor bonding between a fibre and matrix results in poor transverse properties and failures in the form of a fibre pull out, fibre breakage and fibre buckling. Discontinuous fibre reinforced composites have lower strength and stiffness than continuous fibre reinforced composites.

A laminate is a set of laminae stacked to achieve the desired stiffness and thickness. As an example, unidirectional fibre reinforced laminae can be stacked so that the fibres in each lamina are oriented in the same or different directions. The layers are bonded together with the same matrix material. Because of the mismatch of material properties between layers, the shear stresses produced between the layers, especially at the edges of a laminate, may cause delamination [1.11].

3. *REVIEW OF PLATE VIBRATIONS*

The interest of investigators in large vibration amplitudes of plates has been constant since the first revelation of the classical elliptic function solution for simply supported plates by Chu and Herrmann [1.12]. Linear free vibration of composite plates has been studied in the past. The studies of Bert have largely contributed to the development of analytical methods for solution of plate problems [1.13, 1.14]. Reddy [1.15 - 1.18] has reviewed the literature extensively and focused the attention on the application of the finite element method to linear and non-linear plate problems. For composite plates, non-linear strain displacements relationships are most commonly used in the literature for the development of non-linear theories.

Chia[1.19, 1.20] considered the non-linear response of various types of plates. Transverse shear deformation, rotatory inertia, anisotropy, initial imperfections, and variable rigidity have been discussed and reviewed. Developments in free-vibration of analysis of symmetric and unsymmetric laminates, non-linear vibrations of perfect and geometrically imperfect plates have been carried out by Kapania et al. [1.21].

To determine the solutions of the general problem of geometrically non-linear vibrations numerical, analytical or combined analytical numerical methods can be found. The finite element method has been applied to solve non-linear static and dynamic problems of plates. Mei is considered as one of the first researchers to apply the finite element method to large vibrations amplitudes of plates, namely in a work published in 1973 [1.22]. Using the finite element method, Mei et al. [1.23] studied the large amplitude steady state forced vibration response of symmetrically laminated composite thin rectangular plates, including both in-plane deformation and inertia in the formulation.

Most of the research has been carried out in symmetrical composite plates. Asymmetrical laminates are harder to analyse, since they exhibit bending-stretching coupling.

Recent investigations in the geometrically non-linear dynamic behaviour of symmetric laminated plates using the hierarchical finite element method have been developed by W. Han, M. Petyt and P. Ribeiro [1.7-1.9; 1.24-1.28].

In 2002, B. Harras studied the response of rectangular Carbon Fiber Reinforced Plastic (CFRP) and Glare 3 hybrid symmetrically laminated plates in order to investigate the non-linear mode shapes and associated bending stress patterns at large vibration amplitudes of various types of fully clamped rectangular plates. This material offers more resistance to impact than CFRP [1.29].

As referred so far, geometrically non-linear behaviour of asymmetrical laminated plates in composite materials by the hierarchical finite element method is a study that has not been carried out. Based on the model derived in this work, the numerical results obtained will be compared to others in geometrically non-linear behaviour of symmetrical laminated plates in composite materials. Some important conclusions can be found. The first order shear deformation theory is used and compared with results obtained using Kirchooff's hypothesis.

4. *THE HIERARCHICAL FINITE ELEMENT METHOD*

Through the years, structures tended to become more complex. Thus the need to develop new methods to analyse them and the evolution of computers led to the finite element method, used to build non linear models of structures like plates.

Finite Element Analysis (FEA) is a computer-based numerical technique. It can be used to calculate deflection, stress, vibration, buckling behaviour and many other phenomena. It can be used to analyze either small or large-scale deflection under loading or applied displacement. It can analyze elastic deformation, or "permanently bent out of shape" plastic deformation. The computer is required because of the large number of calculations needed to analyze a large structure.

In the finite element method, a structure is broken down into many small simple blocks or elements. The behaviour of an individual element can be described with a relatively simple set of equations. Just as the set of elements would be joined together to build the whole structure, the equations describing the behaviours of the individual elements are joined into a large set of equations (which depends of the structure) that describe the behaviour of the whole structure. The computer can solve this large set of simultaneous equations. From the solution, the computer extracts the behaviour of the individual elements. From this, it can get the stress and deflection of all the parts of the structure. The stresses will be compared to allowed values of stress for the materials to be used, to see if the structure is strong enough [1.30].

The term "finite element" distinguishes the technique from the use of infinitesimal "differential elements" used in calculus, differential equations, and partial differential equations. The method is also distinguished from finite difference equations, for which although the steps into which space is divided are finite in size, there is little freedom in the shapes that the discreet steps can take. Finite element analysis is a way to deal with structures that are more complex than can be dealt with analytically using partial differential equations. FEA deals with complex boundaries better than finite difference equations would, and gives answers to "real world" structural problems. It has been substantially extended in scope during the roughly 40 years of its use.

Finite Element Analysis makes it possible to evaluate a detailed and complex structure, in a computer, during the planning of the structure. The demonstration in the computer of the adequate strength of the structure and the possibility of improving the design during planning can justify the cost of this analysis work. FEA has also been known to increase the rating of structures that were significantly over designed and built many decades ago.

In the absence of Finite Element Analysis (or other numerical analysis), development of structures must be based on experience and hand calculations only. For complex structures, the simplifying assumptions required to make any calculations possible can lead to a conservative and heavy design. A considerable factor of ignorance can remain as to whether the structure will be adequate for all design loads. Significant changes in designs involve risk. Designs will require prototypes to be built and field tested. The field tests may involve expensive strain gauging to evaluate strength and deformation.

In the most used version of the FEM, the shape functions are polynomials with a small degree p, and the accuracy of the model is improved by increasing the number of elements in the structure. As a result, the number of the finite elements increases and their width h decreases, giving to this approach the designation "*h*-version of the FEM".

Another way of improving the accuracy of the finite element approximation is to keep the mesh constant and to increase the number of shape functions over the elements. When polynomials are used as shape functions, this approach implies an increase in their degree p; thus, it was designated as the "*p*-version of the finite element method". If the set of functions corresponding to an approximation of lower order *p*, constitutes a subset of the set of functions corresponding to the approximation of order *p*+1, then the *p*-version of the FEM is called "hierarchical finite element method" (HFEM).

The use of the *p*-version of the finite element method has more advantages than the use of the *h*-version:

i) To achieve more accurate solutions, a change in the mesh is not required;

ii) The linear element matrices for a certain number of shape functions $p=p_1$ are always submatrices for $p=p_2$, $p_2 \geq p_1$[1.31, 1.32].

iii) If orthogonal polynomials are employed, the linear matrices obtained in the hierarchical finite element method are diagonal, thus they are better conditioned than the finite element method matrices [1.30].

iv) the Inclusion Principle, which states that the eigenvalues of the (*n*+1) order approximation bracket the eigenvalues of the *n*th order approximation, is valid for linear discretized systems modelled by the HFEM. Consequently, the HFEM linear solutions converge from above. In general, the Inclusion Principle is not valid for systems modelled by the *h*-version of the FEM [1.32, 1.33].

v) Joining elements of different polynomial degree is not difficult; therefore it is possible to include at low cost additional degrees of freedom where needed [1.31].

vi) Simple structures can be modelled using just one element, or "super-element". This avoids any problems in the satisfaction of inter-element continuity and avoids the assemblage of the elements.

vii) The possibility of choosing the number and type of displacement shape functions facilitates the study of the influence of each displacement component. For example, the influence of the middle plane in-plane displacement components in the dynamic behaviour of a plate can be easily studied.

viii) The HFEM tends to give accurate results with far fewer DOF than the *h*-version of the FEM, because of the flexibility of choosing the shape functions employed according to the problem under study and because of the high order shape functions used. This is particularly true for smooth solutions, since strong mesh generation may be advantageous in the vicinity of singular points [1.34].

As a consequence of these properties, the HFEM model requires less time than the FEM, which is a major advantage in non-linear analysis, where the iterative methods of solution of the equations of motion involve a reformulation of the non-linear matrices in each iteration.

The quick convergence of the HFEM applied in the study of composite laminated plates has been proved [1.7-1.9; 1.24-1.28]. When compared with the *h*-version of the FEM, the HFEM consistently demanded fewer degrees of freedom to accurately calculate stresses, displacements and resonance frequencies.

The large disadvantage of the HFEM is the need to perform integration of high order polynomials which costs many operations in numerical integration. Thus the use of symbolic computation is required. A detailed introduction to the finite element method can be found in references [1.3, 1.32].

5. *OBJECTIVES OF THE PRESENT WORK*

The main objectives of the dissertation that follows this work are:

1. Development of hierarchic finite elements for asymmetrical laminated plates in composite materials. Only symmetric laminates are analysed in this thesis. However, a general model valid for symmetric and asymmetric laminates was derived for wider future use, and this is the model presented in the following text.
2. Analyse the vibration of plates in geometrically non-linear vibrations. To do that, Newmark method will be applied (like finite differences or Wilson-θ, the equations of motion are integrated numerically in time). The analysis will imply the study of the time response, the phase portraits and Fourier spectra. Particular attention will be given to:

- Transitions to chaos (implies the determination of Lyapunov exponents);
- influence of fibre variation.

Both of these objectives will be achieved and are presented in the following chapters.

6. *GENERAL ARRANGEMENT OF CHAPTERS IN THIS THESIS*

This thesis consists of 6 chapters. The first chapter, as has been seen above, is an introduction. The last one contains conclusions which summarize the whole work and suggests future investigations. The mathematical model for the non-linear vibration of composite laminated plates which is based on the Hierarchical Finite Element Method is derived in Chapter 2. In Chapter 3, different tools that can be used to characterize the responses of non-linear systems are presented, namely, Fourier analysis, Poincaré Analysis and Lyapunov exponents. The rest of the chapters are about the applications of the *p*-version finite element model to different cases. By applying the Newmark method to solve the equations of motion the forced vibrations of composite laminated plates due to transverse forces are analysed in Chapter 4. In Chapter 5, in-plane forces are added to the external excitation, and the ensuing motions are discussed.