Chapter 6

CONCLUSIONS AND SUGGESTIONS FOR FUTURE STUDY

1. *CONCLUSIONS*

Composite laminated structures are used in many areas of engineering such as aeronautics, space engineering and naval industry. They are often subjected to large dynamic excitation levels. As a result, they can undergo large amplitude, which cause large tensions and the diminution of life due to fatigue.

In this thesis, the geometrically non-linear forced vibration of fully clamped composite laminated plates was studied. The hierarchical finite element method (HFEM) was used to create the spatial model. The equations of motion for asymmetrically composite laminated plates were derived in the time domain and Newmark direct integration scheme was used to solve them. The implementation was carried out in *FORTRAN*. In the HFEM, particularly in non-linear analysis, high order shape functions must be integrated. The symbolic manipulator *MAPLE* was used to carry out this task, thus defining very accurately the mass and stiffness matrices that constitute the model.

Employing the flexibility of choosing different shape functions to construct the HFEM model, the convergence as a function of the number of shape functions was studied. The number of in-plane shape functions required for accuracy increases as the amplitude of vibration displacement and the non-linearity increase. The use of less in-plane shape functions than the necessary increases the stiffness of the model. For larger amplitudes of vibration, more in-plane than out-of-plane shape functions become necessary. The HFEM gives accurate solutions with far fewer DOF than the *h*-version of the FEM. The influence of b/h was investigated, and as expected, for thin and moderately thick plates, the linear frequencies are lower than for thick plates. Therefore, the thicker the plate, the higher the linear natural frequencies predicted.

The influence of the fibres orientation is studied for a particular plate, considering $\theta = 0^o$, 30^o and 45^o and determining a linear natural frequency parameter. Comparing with other numerical results, the FSDT for thick plates the parameters are lower than those obtained in the classical plate theory (CPT). As the angle increases, the value of the linear natural parameter also increases, which indicates that the fibres orientations influence the linear natural frequency.

In the non-linear analysis, the time domain response for symmetrically laminated composite rectangular plates was obtained for three plates. The force was changed in all the directions and periodic, quasi-periodic and chaotic motions were obtained. In order to analyse solutions, tools such as phase plane, Poincaré maps, Fourier spectra and Lyapunov exponents were used. Periodic, quasi-periodic and chaotic solutions were obtained and the results were discussed. On the other hand, regarding the amplitude of vibration, for all the plates studied, as the force increased, so did the amplitude of vibration.

2. *SUGGESTIONS FOR FUTURE WORK*

Based on the work discussed in the thesis, the following suggestions for future investigation can be made:

i). Once a general model valid for symmetric and asymmetric laminates was derived, work needs to be carried out in asymmetrical plates;

ii). Because of the persistent increasing demands on the performance of mechanical structures and the rising importance of lightweight construction, the mathematical models to describe mechanical systems have to meet increasingly high accuracy. Therefore work can be carried out in the experimental analysis of the plates presented in this thesis;

iii). To obtain the frequency spectrum of quasi-periodic and chaotic oscillations, other tools of signal processing, like the power spectral density function should be implemented;

iv). The algorithm used from Wolf computes the largest non-negative Lyapunov exponent from a time series. The computation of the complete spectrum from the equations of motion, and from its linearization, instead of using the time series needs to be made;

v). The study carried out intended to find non-periodic motion in plates excited harmonically. Thus, academically, some of the amplitudes considered for the forces were very large. As referred before, it will be necessary to validate the linear elastic relationship and the integrity of the plates for very large displacements.