Chapter 5

FORCED VIBRATION OF LAMINATED PLATES - LONGITUDINAL AND TRANSVERSE FORCE

1. *INTRODUCTION*

In the previous chapter, forced vibration analysis in the transverse direction was considered. In this chapter, the force applied to the plate is changed and is applied in the transverse and longitudinal direction. Forces in the plane are constant and compressive. New types of solutions are found. The plates here analysed are given in Tables 1 and 2 of Chapter 4 and the equations of motion are solved using the Newmark method presented in Chapter 2. In order to analyse the time domain response of the plate's vibrations, the tools presented in Chapter 3 are used to determine the presence of a periodic, quasi-periodic or chaotic motion.

2. *NON-LINEAR FORCED VIBRATION ANALYSIS*

2.1 - *Distributed Applied Force*

In this section, the forced vibration of a rectangular plate is studied using the HFEM, and equation (2.71) is solved by the Newmark method. When a distributed load in the transverse and longitudinal direction is applied, the generalized forces are given by equation (2.77) in Chapter 2.

2.2 - *Numerical results*

For plate 2, and for a fibre orientation of $\theta = 45^{o}$, two cases are considered: in the first case, the plate is excited at 5000 *N/*m^2 in the *x* direction, 7000 *N/*m^2 in the *y* direction and in the *z* direction the force varies from 500 *N/*m^2 to 7000 *N/*m^2 for an excitation frequency of 762.888 *rad/s*; in the second case, the forces in the *x, y* and *z* directions are kept at 10000 *N/*m^2 but the frequency of excitation is changed from 762.888 *rad/s* to 900 *rad/s*; 114 DOF (pi=7, p_o=4, p_θ=7) are considered in the model, and the results are discussed. In the first case, the damping factor, α, is equal to 0.00001 and in the second is 0.000001.

For plate 3, two other cases are studied: in the first case, the forces in the *x*, *y*, *z* directions are equal and are increased from 15000 *N/*m^2 to 100000 *N/*m^2; in the second case the plate is excited at 5000 *N/*m^2 in the *x* direction, 7000 *N/*m^2 in the *y* direction and in the *z* direction the force varies from 7500 *N/*m^2 to 50000 *N/*m^2. In the first case, the damping factor, α, is equal to 0.00001 and in the second is 0.000001.

In both cases the excitation frequency is 980.592 *rad/s*. The results obtained are also discussed. For both plates, the damping factor, α, is equal to 0.00001.

In Figure 1, the time domain response and the phase plane of plate 2 are presented for a force of 5000 *N/*m^2 in the *x* direction, 7000 *N/*m^2 in the *y* direction and 500 *N/*m^2 in the *z* direction.

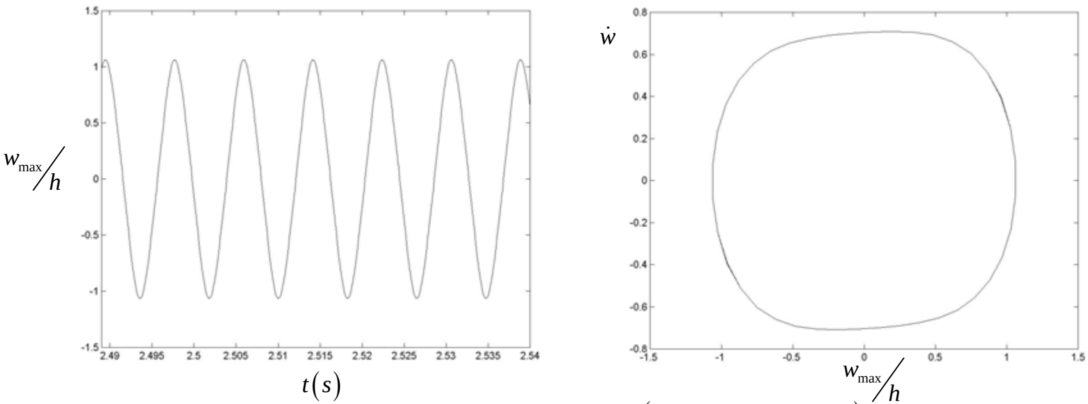

Figure 1 - Time history and phase plane of Plate 2, $(45,-45,45,-45,45)$ due to excitation of $(F_x, F_y, F_z) = (5000, 7000, 500)$ *N/m*^{2}

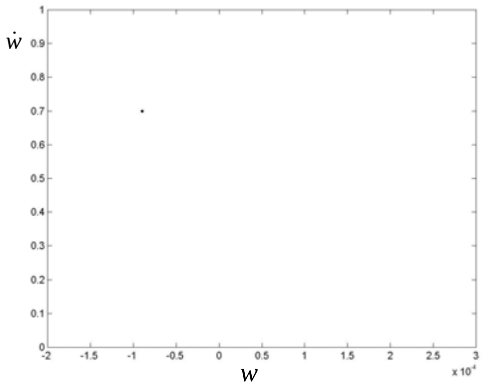

Figure 2 - Poincaré map of Plate 2, due to excitation $(F_x, F_y, F_z) = (5000, 7000, 500)$ N/m²

From Figures 1 and 2, one sees that for $(F_x, F_y, F_z) = (5000, 7000, 500)$ *N/m²*, a periodic solution is obtained. In fact, the phase portrait is closed, and the Poincaré map tends to a point.

In Figures 3 to 8, the force in the *z* direction is increased to 7000 *N/m²*.

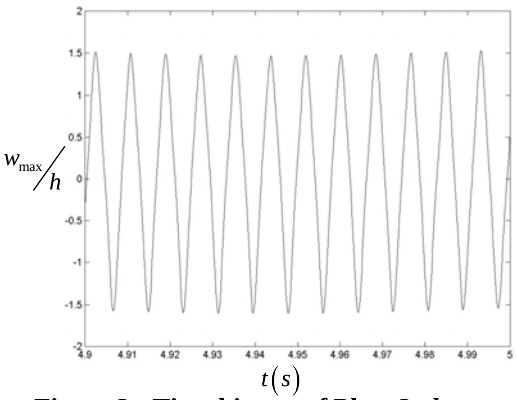

Figure 3 - Time history of Plate 2, due to excitation $(F_x, F_y, F_z) = (5000, 7000, 1000)$ N/m²

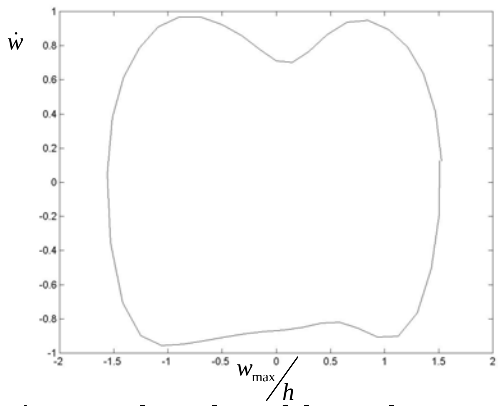

Figure 4 - Phase plane of the steady state forced vibration $(F_x, F_y, F_z) = (5000, 7000, 1000)$ N/m² of plate 2

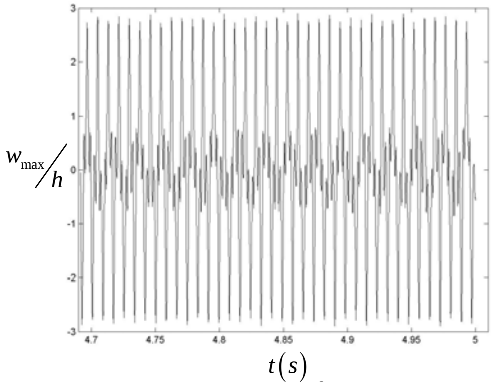

Figure 5 - Time history of Plate 2, due to excitation $\left(F_x, F_y, F_z\right) = \left(5000, 7000, 3000\right)$ N/m²

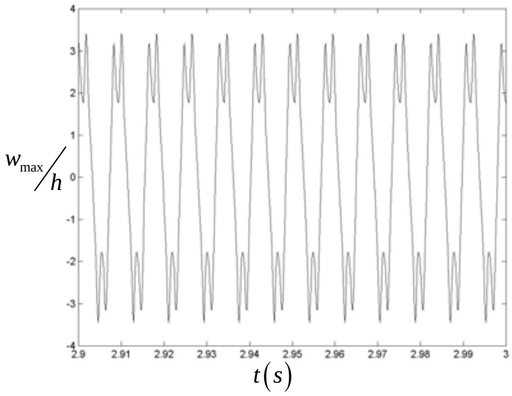

Figure 7 - Time history of Plate 2, due to excitation $\left(F_x, F_y, F_z\right) = \left(5000, 7000, 7000\right)$ N/m²

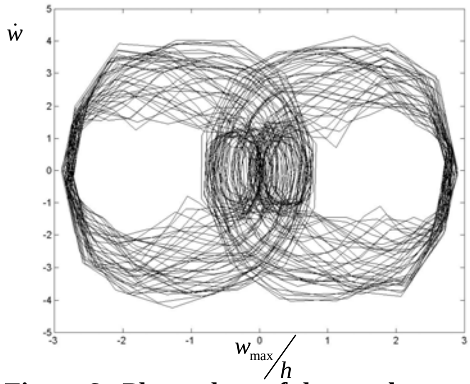

Figure 6 - Phase plane of the steady state forced vibration $\left(F_x, F_y, F_z\right) = \left(5000, 7000, 3000\right)$ N/m² of plate 2

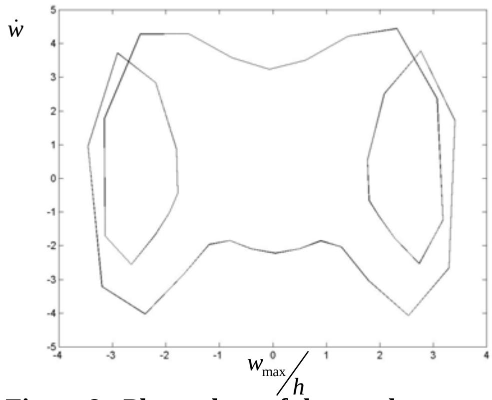

Figure 8 - Phase plane of the steady state forced vibration $\left(F_x, F_y, F_z\right) = \left(5000, 7000, 7000\right)$ N/m² of plate 2

From Figures 3 to 8, different types of solutions are found. The Poincaré maps of these responses were computed and are presented in Figures 9 to 11.

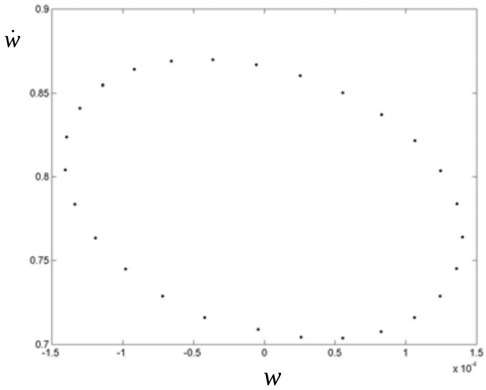

Figure 9 - Poincaré map of Plate 2, due to excitation $\left(F_x, F_y, F_z\right) = \left(5000, 7000, 1000\right)$ N/m²

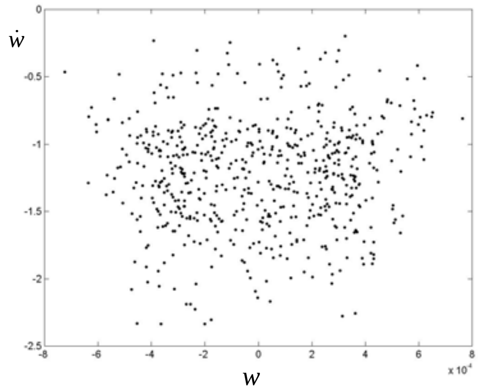

Figure 10 - Poincaré map of Plate 2, due to excitation $\left(F_x, F_y, F_z\right) = \left(5000, 7000, 3000\right)$ N/m²

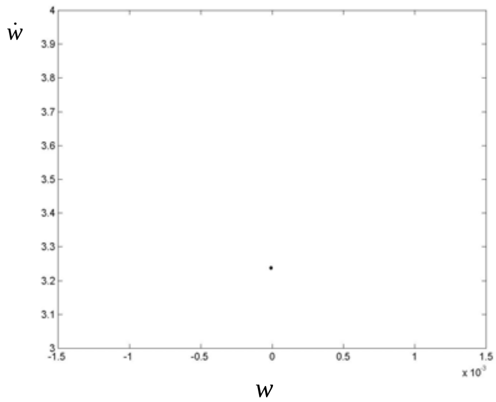

Figure 11 - Poincaré map of Plate 2, due to excitation $\left(F_x, F_y, F_z\right) = \left(5000, 7000, 7000\right)$ N/m²

For a force of 1000 N/m^2 in the z direction, once an closed path is obtained in the phase plane(Figure 4) and the Poincaré map (Figure 9) tends to a closed line, a quasi-periodic solution was found; for $F_z = 3000$ N/m^2 (Figure 10), a cloud of points in the Poincaré map may represent a chaotic solution (to be confirmed with the computation of Lyapunov exponents); in Figure 11, once a single point is obtained in the Poincaré map, a periodic solution is obtained. In Figure 12 the Fourier spectrum for the excitation $\left(F_x, F_y, F_z\right) = \left(5000, 7000, 7000\right)$ is presented.

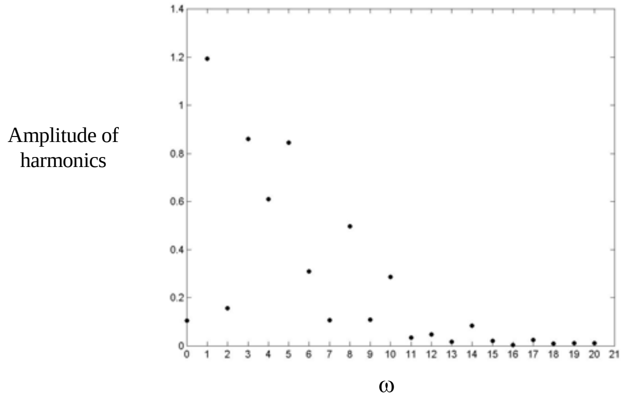

Figure 12 - Fourier spectrum of forced of Plate 2, due to excitation $\left(F_x, F_y, F_z\right) = \left(5000, 7000, 7000\right)$ N/m²

Figure 12 confirms the results obtained in the Poincaré map. For $F_z = 7000$ *N/m²* a periodic solution is found, where harmonics of the excitation frequency are involved. In Figure 13, a positive Lyapunov exponent is obtained therefore a chaotic motion is achieved for $F_z = 3000$ *N/m²*.

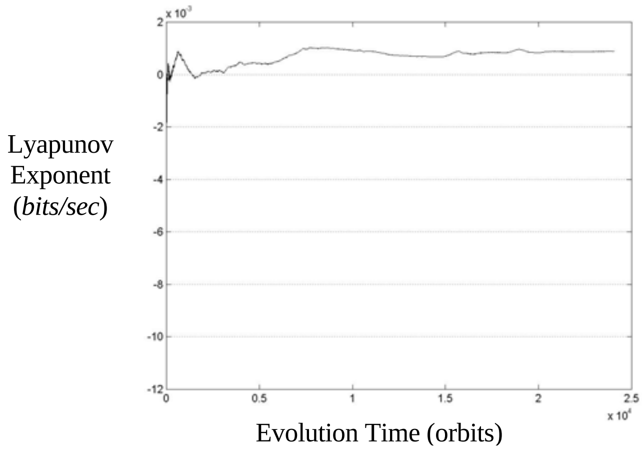

Figure 13 – Highest Lyapunov exponent for Plate 2, due to excitation $\left(F_x, F_y, F_z\right) = \left(5000, 7000, 3000\right)$ N/m²

The following case study is plate 2, where the forces in the *x*, *y* and *z* directions are kept at 10000 *N/*m^2 but the frequency of excitation is changed from 762.888 *rad/s* to 900 *rad/s*.

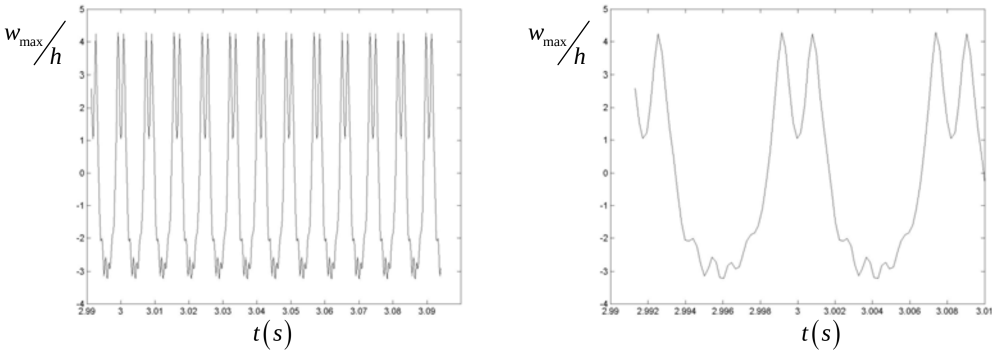

Figure 14 - Time history of Plate 2, due to excitation $(F_x, F_y, F_z) = (10000, 10000, 10000)$ N/m^2, $\omega = 762.888$ *rad/s*

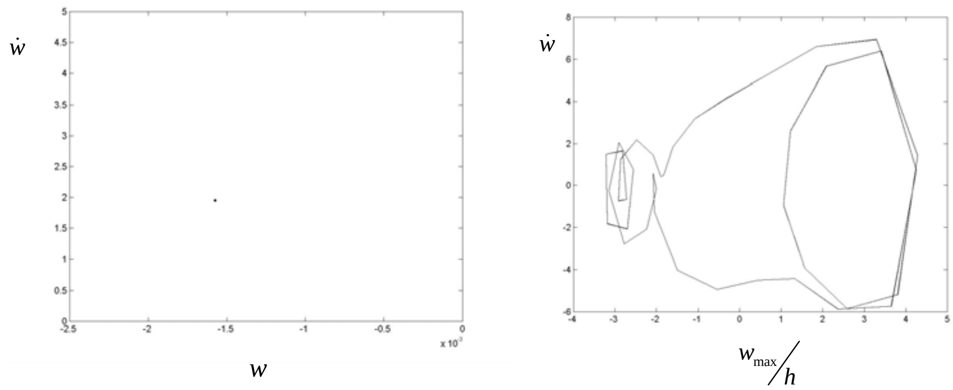

Figure 15 - Poincaré map and phase plane of the steady state forced vibration $(F_x, F_y, F_z) = (10000, 10000, 10000)$ N/m^2 of plate 2, $\omega = 762.888$ *rad/s*

In Figures 14 and 15, a closed path is obtained in the phase plane and the Poincaré map consists of a single point, therefore a periodic solution is found.

Considering $\omega = 800$ *rad/s* and $\omega = 900$ *rad/s*, the time domain responses and the phase plane are given in Figures 16 to 19:

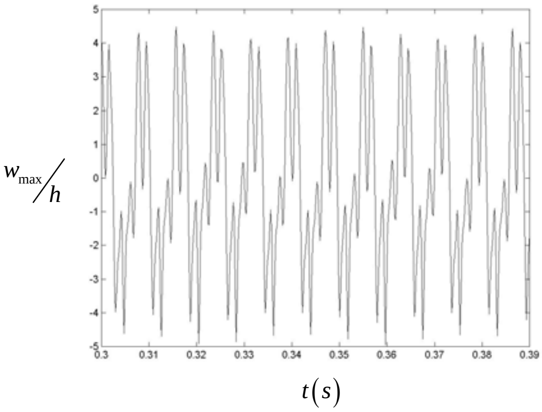

Figure 16 – Time history of Plate 2, due to excitation
$(F_x, F_y, F_z) = (10000, 10000, 10000)$ **N/m²,**
$\omega = 800$ ***rad/s***

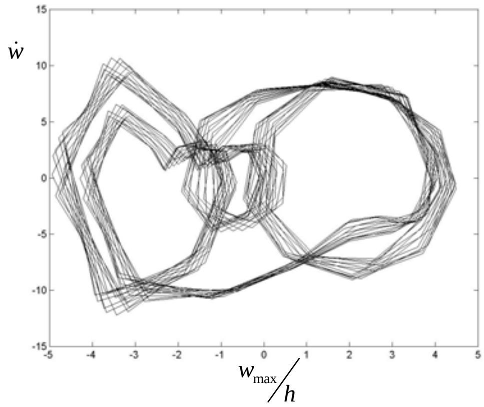

Figure 17 – Phase Plane of Plate 2, due to excitation
$(F_x, F_y, F_z) = (10000, 10000, 10000)$ **N/m²,**
$\omega = 800$ ***rad/s***

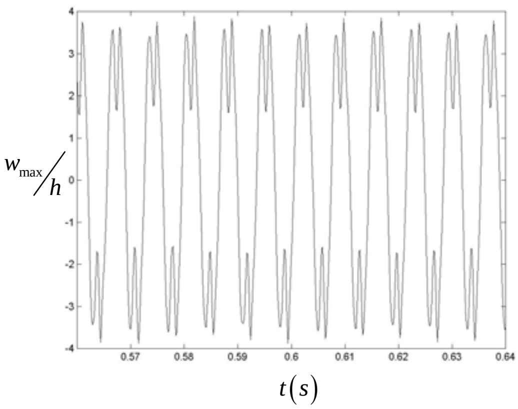

Figure 18 - Time domain response of Plate 2, due to excitation
$(F_x, F_y, F_z) = (10000, 10000, 10000)$ **N/m²,**
$\omega = 900$ ***rad/s***

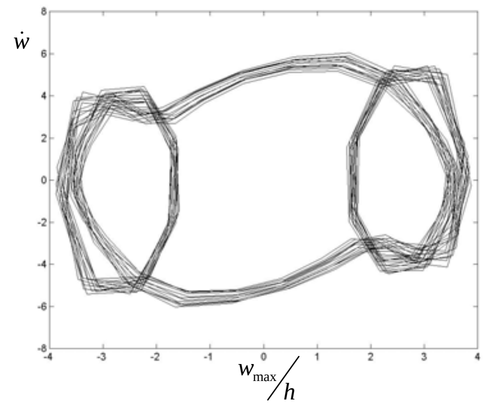

Figure 19 - Phase Plane of Plate 2, due to excitation
$(F_x, F_y, F_z) = (10000, 10000, 10000)$ **N/m²,**
$\omega = 900$ ***rad/s***

It is difficult to ascertain, from the phase planes, if the path is closed or not. Apparently it is not, meaning that the solution is not periodic. This is - again apparently - confirmed by the Poincaré maps in Figures 20 and 21, although more points would be necessary to distinguish between a periodic motion with a large period or a not periodic motion.

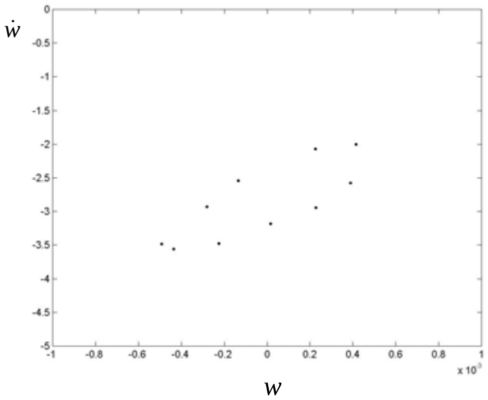

Figure 20 – Poincaré Map of Plate 2, due to excitation $\left(F_x, F_y, F_z\right) = \left(10000, 10000, 10000\right)$ N/m², $\omega = 800$ *rad/s*

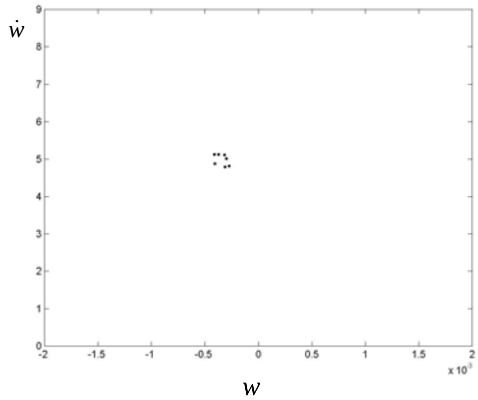

Figure 21 - Poincaré Map of Plate 2, due to excitation $\left(F_x, F_y, F_z\right) = \left(10000, 10000, 10000\right)$ N/m², $\omega = 900$ *rad/s*

In Figures 22 to 39, the time domain response, the phase plane and the Poincaré map of plate 3 are presented for a force increased from 15000 N/m^2 to 100000 N/m^2 in *x*, *y*, *z* directions and $\omega = 980.592$ *rad/s*.

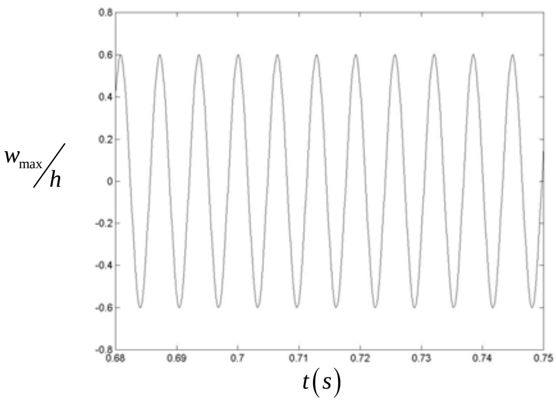

Figure 22 - Time history of Plate 3, due to excitation $\left(F_x, F_y, F_z\right) = \left(15000, 15000, 15000\right)$ N/m², $\omega = 980.592$ *rad/s*

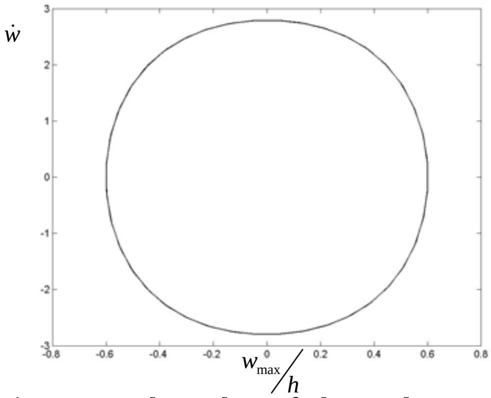

Figure 23 – Phase plane of Plate 3, due to excitation
$\left(F_x, F_y, F_z\right) = \left(15000, 15000, 15000\right)$ **N/m²,**
$\omega = 980.592$ ***rad/s***

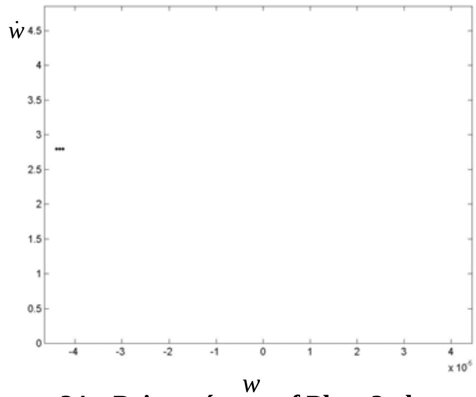

Figure 24 – Poincaré map of Plate 3, due to excitation
$\left(F_x, F_y, F_z\right) = \left(15000, 15000, 15000\right)$ **N/m²,**
$\omega = 980.592$ ***rad/s***

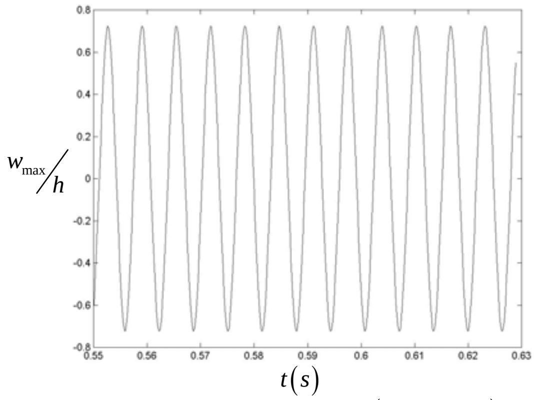

Figure 25 - Time history of Plate 3, due to excitation $\left(F_x, F_y, F_z\right) = \left(20000, 20000, 20000\right)$ **N/m²,**
$\omega = 980.592$ ***rad/s***

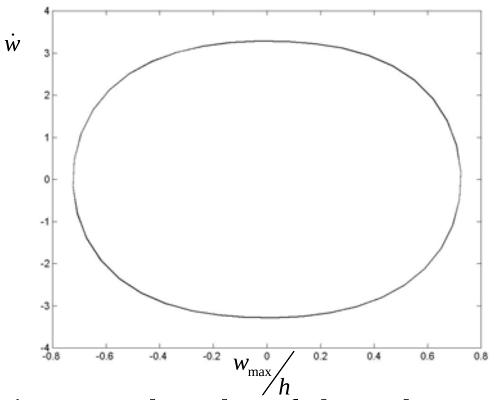

Figure 26 – Phase plane of Plate 3, due to excitation
$\left(F_x, F_y, F_z\right) = \left(20000, 20000, 20000\right)$ **N/m²,**
$\omega = 980.592$ ***rad/s***

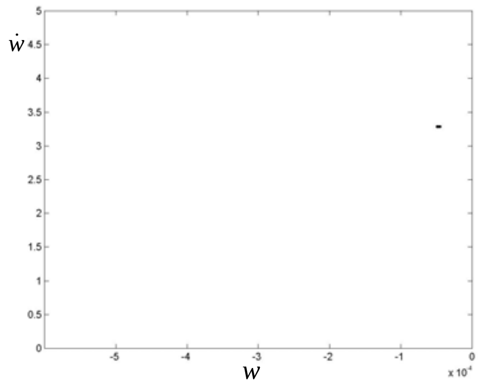

Figure 27 -Poincaré of Plate 3, due to excitation
$\left(F_x, F_y, F_z\right) = \left(20000, 20000, 20000\right)$ **N/m²,**
$\omega = 980.592$ ***rad/s***

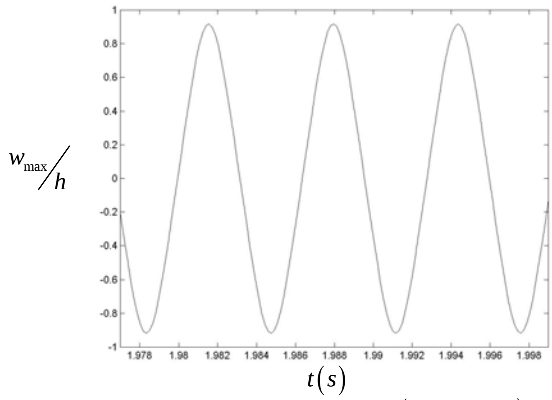

Figure 28 - Time history of Plate 3, due to excitation $\left(F_x, F_y, F_z\right) = \left(30000, 30000, 30000\right)$ N/m^2, $\omega = 980.592$ *rad/s*

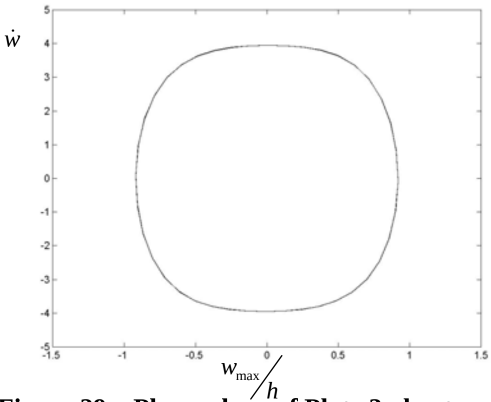

Figure 29 – Phase plane of Plate 3, due to excitation $\left(F_x, F_y, F_z\right) = \left(30000, 30000, 30000\right)$ N/m^2, $\omega = 980.592$ *rad/s*

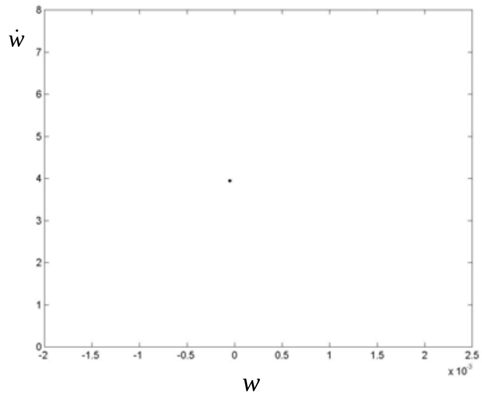

Figure 30 –Poincaré map of Plate 3, due to excitation $\left(F_x, F_y, F_z\right) = \left(30000, 30000, 30000\right)$ N/m^2, $\omega = 980.592$ *rad/s*

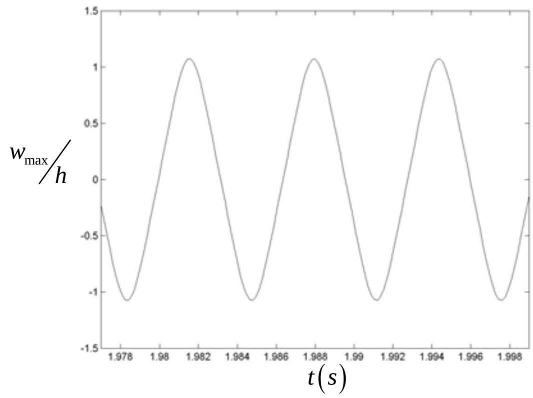

Figure 31 - Time history of Plate 3, due to excitation $\left(F_x, F_y, F_z\right) = \left(40000, 40000, 40000\right)$ N/m^2, $\omega = 980.592$ *rad/s*

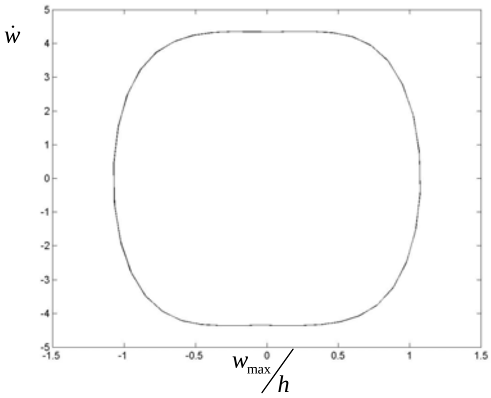

Figure 32 – Phase plane of Plate 3, due to excitation $(F_x, F_y, F_z) = (40000, 40000, 40000)$ N/m^2, $\omega = 980.592$ *rad/s*

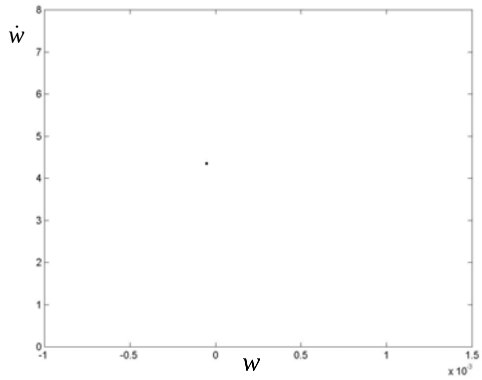

Figure 33 – Poincaré map of Plate 3, due to excitation $(F_x, F_y, F_z) = (40000, 40000, 40000)$ N/m^2, $\omega = 980.592$ *rad/s*

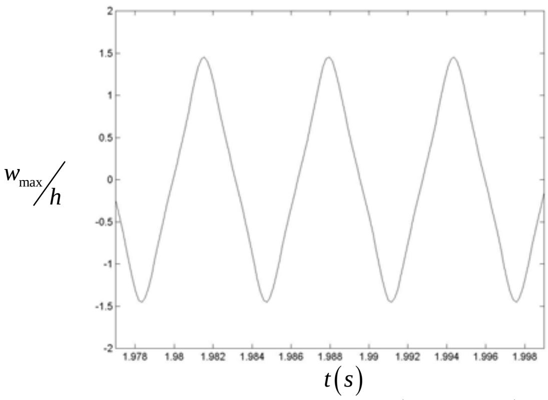

Figure 34 - Time history of Plate 3, due to excitation $(F_x, F_y, F_z) = (70000, 70000, 70000)$ N/m^2, $\omega = 980.592$ *rad/s*

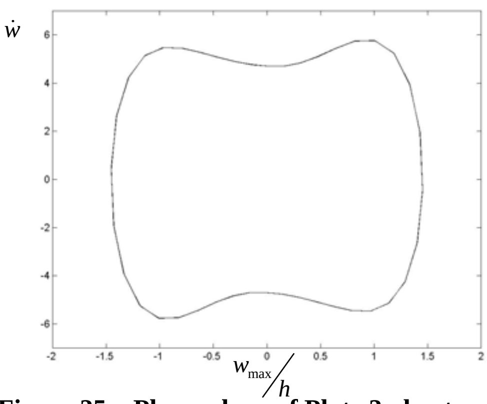

Figure 35 – Phase plane of Plate 3, due to excitation $(F_x, F_y, F_z) = (70000, 70000, 70000)$ N/m^2, $\omega = 980.592$ *rad/s*

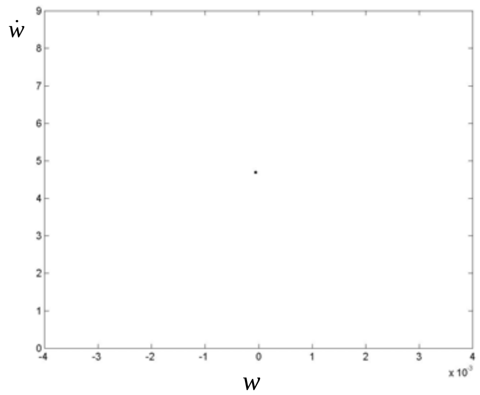

Figure 36 – Poincaré map of Plate 3, due to excitation $(F_x, F_y, F_z) = (70000, 70000, 70000)$ N/m^2, $\omega = 980.592$ *rad/s*

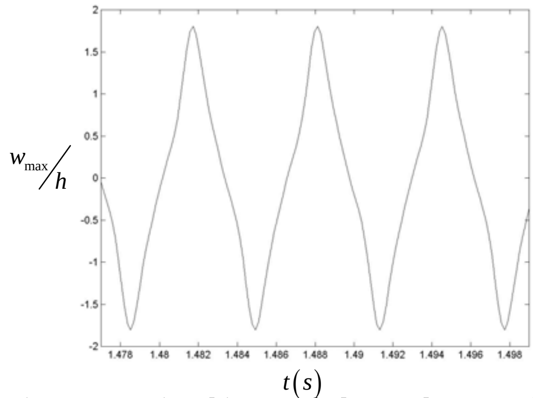

Figure 37 – Time history of Plate 3, due to excitation
$\left(F_x, F_y, F_z\right) = \left(100000, 100000, 100000\right)$ **N/m^2,** $\omega = 980.592$ ***rad/s***

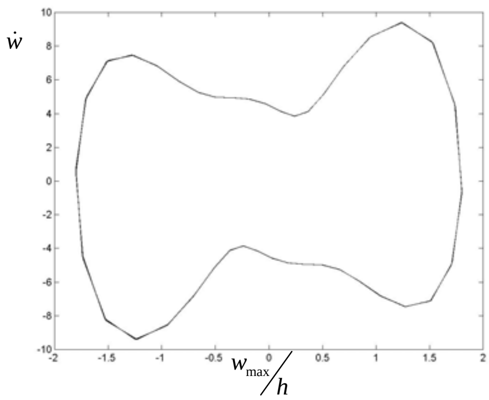

Figure 38 – Phase plane of Plate 3, due to excitation
$\left(F_x, F_y, F_z\right) = \left(100000, 100000, 100000\right)$
N/m^2, $\omega = 980.592$ ***rad/s***

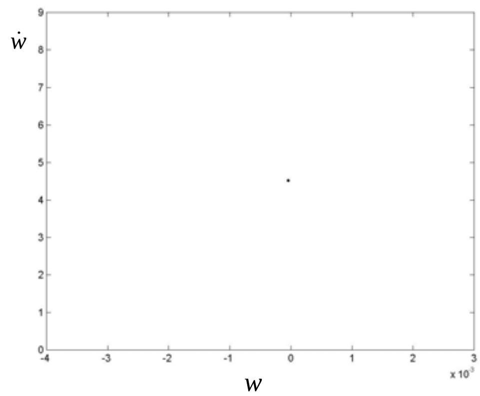

Figure 39 – Poincaré map of Plate 3, due to excitation
$\left(F_x, F_y, F_z\right) = \left(100000, 100000, 100000\right)$
N/m^2, $\omega = 980.592$ ***rad/s***

In all the cases presented above, the time domain response represents a periodic solution. As the force increases, so does the amplitude of vibration, as expected.

For the second case of plate 3, in Figures 40 to 60, the force is increased from 7500 *N/m^2* to 50000 *N/m^2* in the *z* direction and $\left(F_x, F_y\right) = \left(5000, 7000\right)$ *N/m^2*.

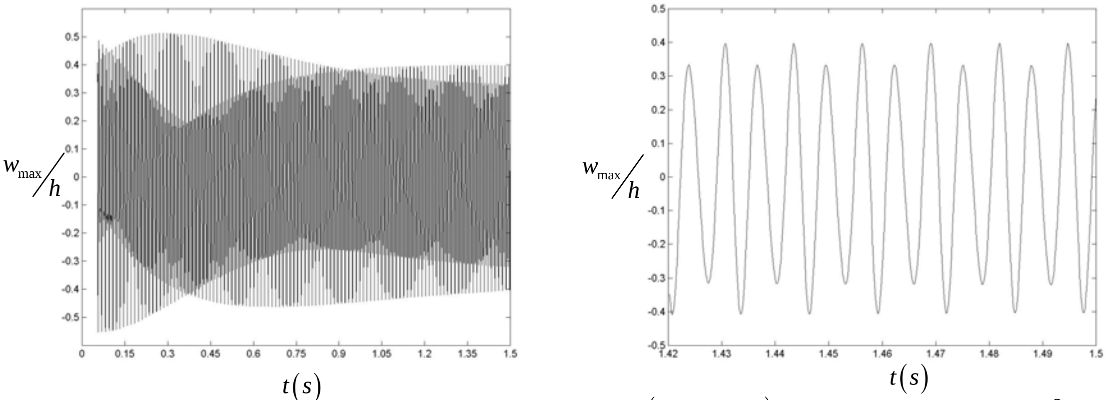

Figure 40 – Time history of Plate 3, due to excitation $(F_x, F_y, F_z) = (5000, 7000, 7500)$ N/m^2, $\omega = 980.592$ *rad/s*

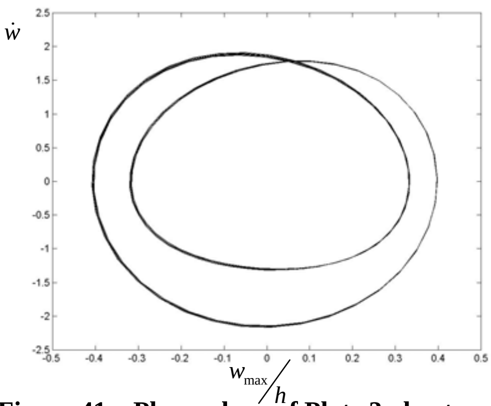

Figure 41 – Phase plane of Plate 3, due to excitation $(F_x, F_y, F_z) = (5000, 7000, 7500)$ N/m^2, $\omega = 980.592$ *rad/s*

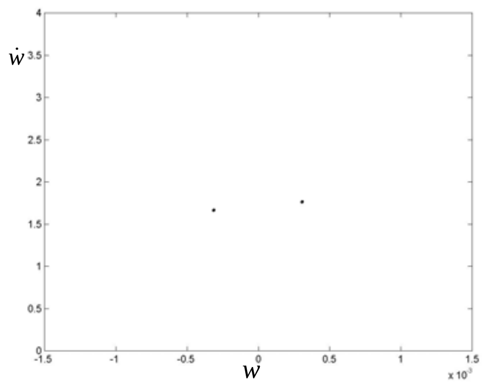

Figure 42 – Poincaré map of Plate 3, due to excitation $(F_x, F_y, F_z) = (5000, 7000, 7500)$ N/m^2, $\omega = 980.592$ *rad/s*

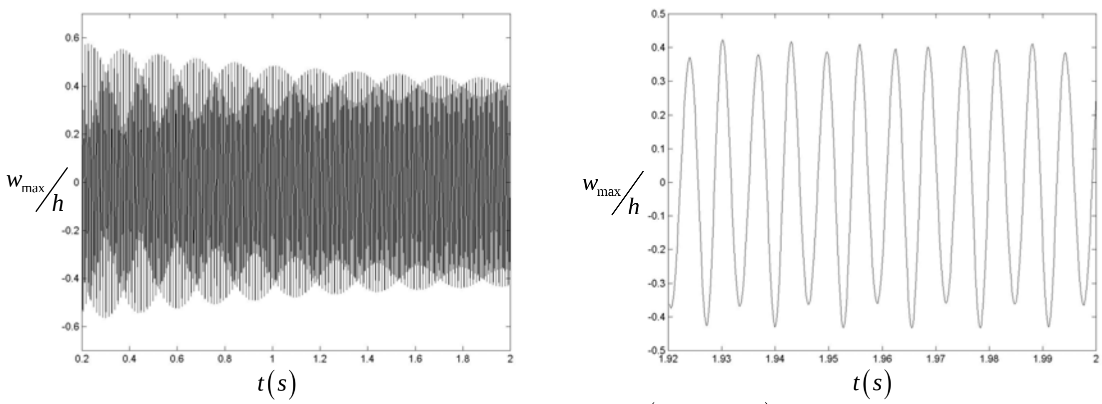

Figure 43 – Time history of Plate 3, due to excitation $(F_x, F_y, F_z) = (5000, 7000, 8500)$ N/m^2, $\omega = 980.592$ *rad/s*

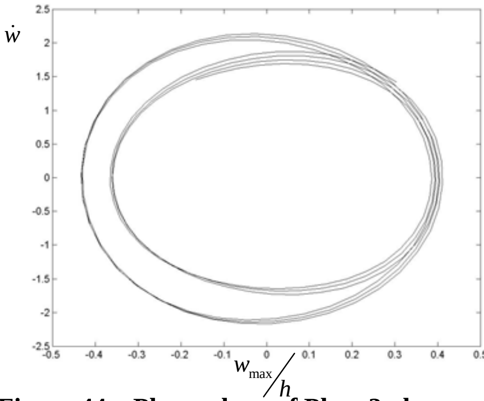

Figure 44 – Phase plane of Plate 3, due to excitation $\left(F_x, F_y, F_z\right) = \left(5000, 7000, 8500\right)$ N/m², ω = 980.592 *rad/s*

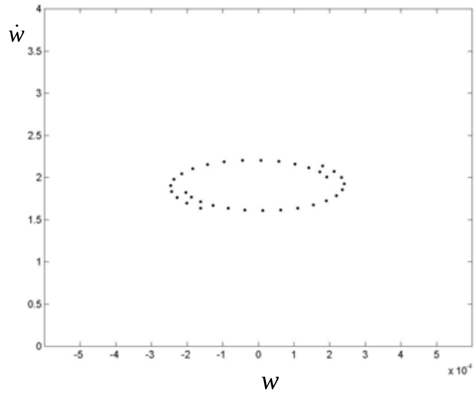

Figure 45 – Poincaré map of Plate 3, due to excitation $\left(F_x, F_y, F_z\right) = \left(5000, 7000, 8500\right)$ N/m², ω = 980.592 *rad/s*

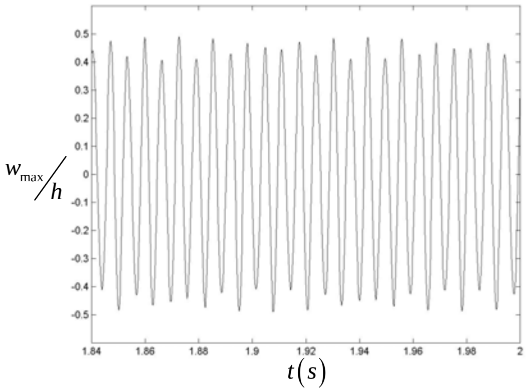

Figure 46 – Time history of Plate 3, due to excitation $\left(F_x, F_y, F_z\right) = \left(5000, 7000, 10000\right)$ N/m², ω = 980.592 *rad/s*

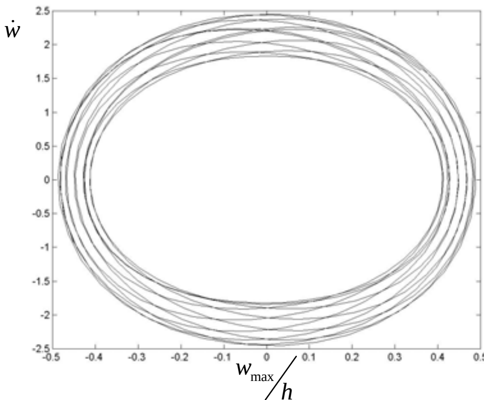

Figure 47 – Phase plane of Plate 3, due to excitation $\left(F_x, F_y, F_z\right) = \left(5000, 7000, 10000\right)$ N/m², ω = 980.592 *rad/s*

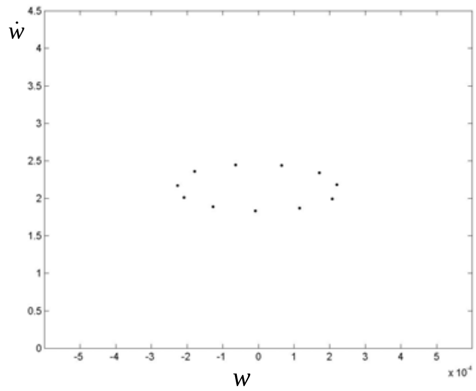

Figure 48 – Poincaré map of Plate 3, due to excitation $\left(F_x, F_y, F_z\right) = \left(5000, 7000, 10000\right)$ N/m², ω = 980.592 *rad/s*

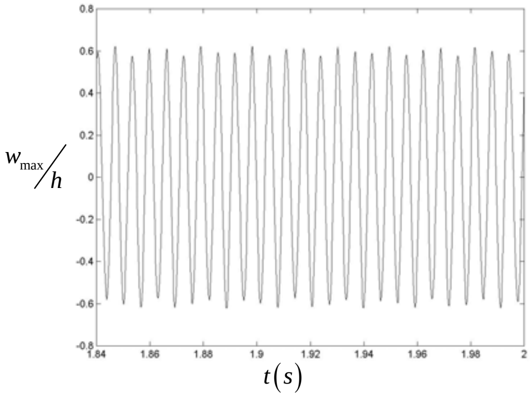

Figure 49 – Time history of Plate 3, due to excitation $(F_x, F_y, F_z) = (5000, 7000, 15000)$ N/m^2, ω = 980.592 *rad/s*

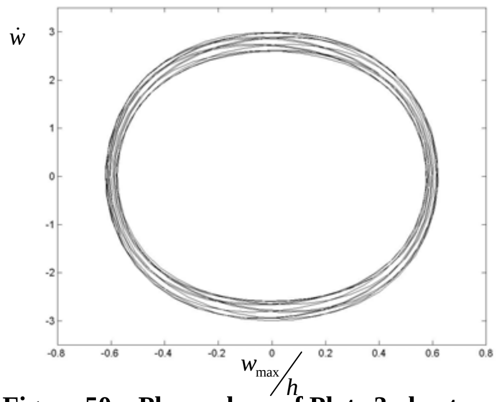

Figure 50 – Phase plane of Plate 3, due to excitation $(F_x, F_y, F_z) = (5000, 7000, 15000)$ N/m^2, ω = 980.592 *rad/s*

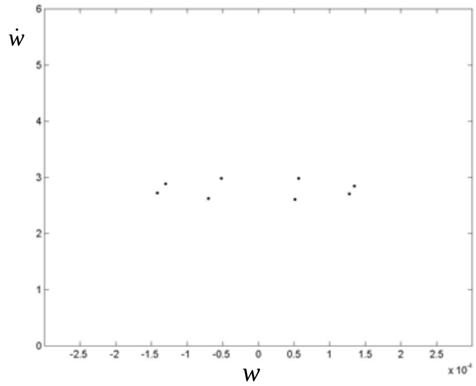

Figure 51 – Poincaré map of Plate 3, due to excitation $(F_x, F_y, F_z) = (5000, 7000, 15000)$ N/m^2, ω = 980.592 *rad/s*

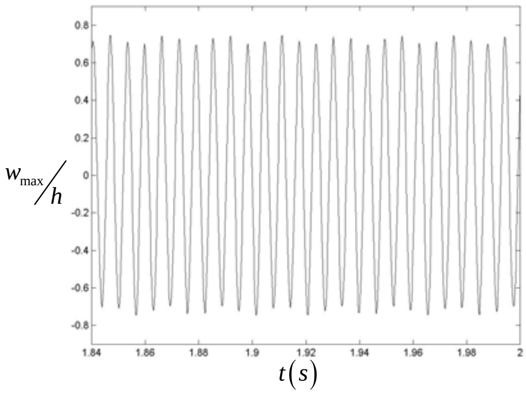

Figure 52 – Time history of Plate 3, due to excitation $(F_x, F_y, F_z) = (5000, 7000, 20000)$ N/m^2, ω = 980.592 *rad/s*

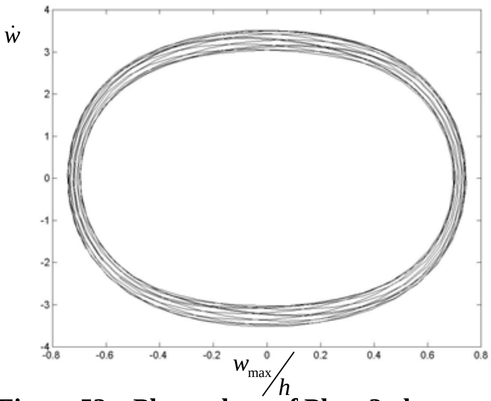

Figure 53 – Phase plane of Plate 3, due to excitation
$\left(F_x, F_y, F_z\right) = \left(5000, 7000, 20000\right)$ **N/m²,**
$\omega = 980.592$ ***rad/s***

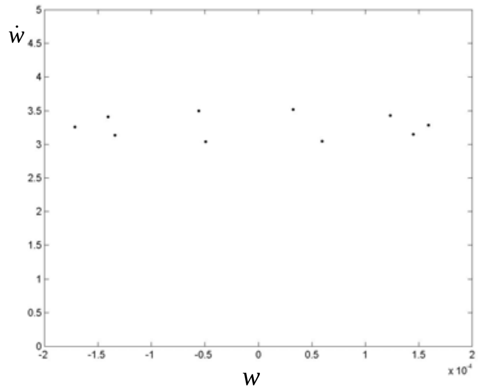

Figure 54 – Poincaré map of Plate 3, due to excitation
$\left(F_x, F_y, F_z\right) = \left(5000, 7000, 20000\right)$ **N/m²,**
$\omega = 980.592$ ***rad/s***

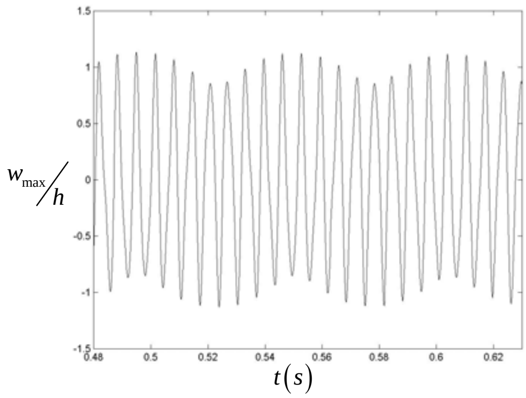

Figure 55 – Time history of Plate 3, due to excitation $\left(F_x, F_y, F_z\right) = \left(5000, 7000, 35000\right)$ **N/m²,**
$\omega = 980.592$ ***rad/s***

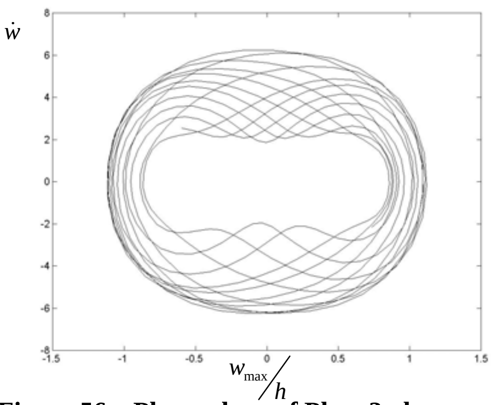

Figure 56 – Phase plane of Plate 3, due to excitation
$\left(F_x, F_y, F_z\right) = \left(5000, 7000, 35000\right)$ **N/m²,**
$\omega = 980.592$ ***rad/s***

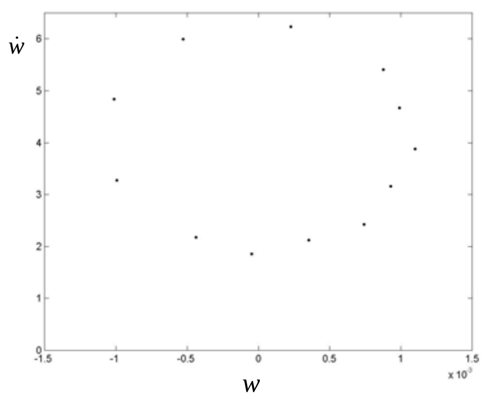

Figure 57 – Poincaré map of Plate 3, due to excitation
$\left(F_x, F_y, F_z\right) = \left(5000, 7000, 35000\right)$ **N/m²,**
$\omega = 980.592$ ***rad/s***

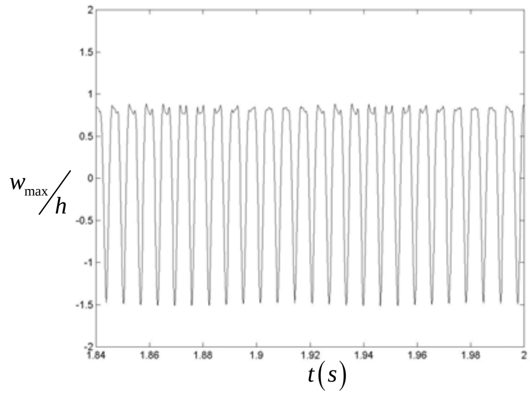

Figure 58 – Time history of Plate 3, due to excitation $\left(F_x, F_y, F_z\right) = \left(5000, 7000, 50000\right)$ N/m^2, $\omega = 980.592$ *rad/s*

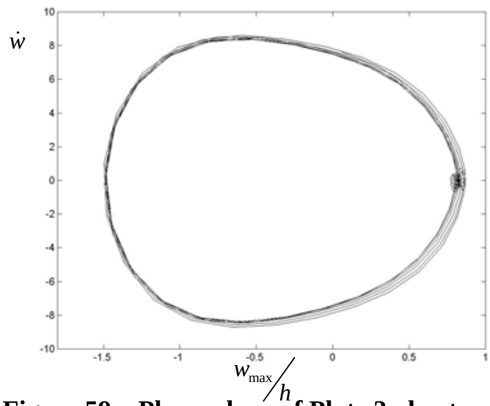

Figure 59 – Phase plane of Plate 3, due to excitation $\left(F_x, F_y, F_z\right) = \left(5000, 7000, 50000\right)$ N/m^2, $\omega = 980.592$ *rad/s*

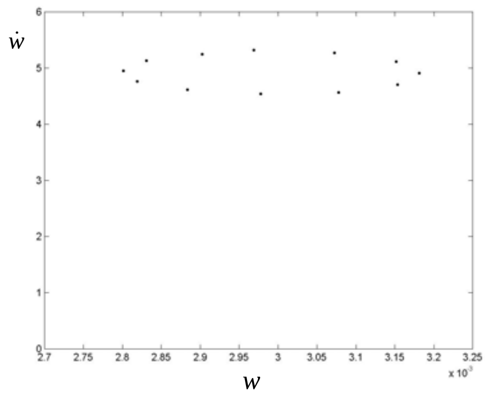

Figure 60 – Poincaré map of Plate 3, due to excitation $\left(F_x, F_y, F_z\right) = \left(5000, 7000, 50000\right)$ N/m^2, $\omega = 980.592$ *rad/s*

From the figures above, one sees that the response to harmonic, transverse excitations, in the presence of in-plane constant forces, can be of several types: periodic, but almost harmonic; periodic, with strong influence of harmonics of the excitation frequency; quasi-periodic and chaotic. To remove all doubts about the possibly quasi-periodic motions it would be necessary to compute the two largest Lyapunov exponents [5.1]

3. *CONCLUSIONS*

In this chapter, forced vibrations of composite laminated plates modelled by the HFEM are analysed. The main difference from Chapter 4 is the change of the force applied to the plate.

For plate 2, two cases are considered: in the first case, the plate is excited at 5000 N/m^2 in the x direction, 7000 N/m^2 in the y direction and in the z direction the force varies from 500 N/m^2 to 7000 N/m^2 for an excitation frequency of 762.888 *rad/s*; in the second case, the forces in the x, y and z directions are kept at 10000 N/m^2 but the frequency of excitation is changed from 762.888 *rad/s* to 900 *rad/s*. For the first case as the force in the z direction increases, so does the amplitude of vibration. Periodic, quasi-periodic and chaotic motions are obtained and the results are confirmed with the computation of Poincaré maps, Fourier spectrum and Lyapunov exponents. In the second case, increasing the excitation frequency, quasi-periodic solutions are obtained and the amplitude of vibration diminishes as the excitation frequency increases.

For plate 3, two other cases are considered: in the first case, the forces in the x, y, z directions are equal and are increased from 15000 N/m^2 to 100000 N/m^2; in the second case the plate is excited at 5000 N/m^2 in the x direction, 7000 N/m^2 in the y direction and in the z direction the force varies from 7500 N/m^2 to 50000 N/m^2. In both cases the excitation frequency is 980.592 *rad/s*. In the first case, as the force in all the directions increase, so does the amplitude of vibration. Once a closed line is obtained in the phase plane and the Poincaré map consists of a single point, a periodic solution is obtained. For the second case, only when the plate is excited with a force of 7500 N/m^2, a periodic solution is obtained. In all the other responses, possible quasi-periodic solutions are obtained. To remove all doubts it would be necessary to compute the two largest Lyapunov exponents.

It must be mentioned, that it was assumed that the plate always vibrate in the linear elastic regime. Naturally, for engineering applications it ought to be confirmed if the elastic limit is not passed. The same is true in what delamination is concerned.