

FROM PERIODIC TO CHAOTIC OSCILLATIONS IN COMPOSITE LAMINATED  
PLATES

P. Ribeiro

IDMEC/DEMEGI,

Faculty of Engineering, University of Porto

R. Dr. Roberto Frias, s/n, 4200-465 Porto, Portugal

Fax: + 351 22 508 14 45; Tel: +351 22 508 17 13; e-mail: pmleal@fe.up.pt

R.P. Duarte

Department of Informatics, Engineering Superior School of Viseu, Viseu, Portugal

ABSTRACT

The geometrically non-linear, linear elastic, oscillations of composite laminated plates are studied in the time domain by direct numeric integration of the equations of motion. A  $p$ -version finite element recently proposed for moderately thick plates is employed to define the mathematical model. By applying transverse harmonic forces, the variation of the oscillations with the angle of the fibres is investigated. In this case, only periodic motions with a period equal to the excitation are found. However, introducing in-plane forces,  $m$ -periodic or quasi-periodic, and chaotic oscillations are defined. The existence of chaos is confirmed by computing the largest Lyapunov exponent.

Keywords: laminated plates,  $p$ -version, non-linear, dynamics, chaos

## 1. INTRODUCTION

Composite laminated plates are used in many areas such as aeronautics, naval industry and microelectronics [Erro! A origem da referência não foi encontrada.]. If actuated by large excitations at frequencies close to resonance frequencies, those plates may undergo vibrations with large amplitude, therefore in the geometrically non-linear regime. In this case, it has been found that harmonic excitations can produce periodic oscillations where harmonics of the excitation frequency are quite important [Erro! A origem da referência não foi encontrada.-Erro! A origem da referência não foi encontrada.].

Quasi-periodic and chaotic oscillations are also possible in composite laminates. In reference [Erro! A origem da referência não foi encontrada.] one composite plate and one composite cylinder are investigated by numerically integrating in the time domain a set of finite element equations. The excitation is provided by a temperature field and the response described by time and phase plots. According to the authors, the highly irregular oscillations of the plate observed after a certain number of cycles - when displacements as large as 15 times the plate thickness are attained - are chaotic due to their irregularity. In fact, the portrayed motions appear to be quite irregular from the outset, *i.e.*, even for low vibration amplitudes. Maestrello presents in reference [Erro! A origem da referência não foi encontrada.] an experimental analysis on two adjacent aircraft panels forced by a turbulent boundary layer and a pure tone sound. Responses changing from periodic, to quasi-periodic and finally to chaotic - in a characteristic route to chaos [Erro! A origem da referência não foi encontrada.] - were observed. To describe the type of motion, the time histories, spectral densities and

phase portraits were presented and the largest Lyapunov exponent computed for some experimental data. Another noteworthy study is presented in reference **[Erro! A origem da referência não foi encontrada.]**, where the behaviour of a laminated plate under the effects of high-supersonic flow was analysed. The reduced set of equations of motion, which was obtained applying Galerkin's method, was solved by a continuation procedure to determine static solutions; the shooting and multiple-scales methods were employed to find periodic motions and direct numerical integration for non-periodic states.

Accurate numerical prediction of multi-harmonic, quasi-periodic or chaotic oscillations of a structure is difficult to undertake essentially due to two reasons, which contribute to large computational time and memory requirements. First, for most cases, an accurate model requires a large number of degrees of freedom, particularly if one takes into account that the non-linear mode shape is not constant and that several modes may influence the response. Second, higher frequencies are likely to appear in the response, therefore a time domain analysis requires the use of small time steps.

A  $p$ -version, hierarchical finite element was developed recently for moderately thick laminated plates and applied to study periodic free vibrations **[Erro! A origem da referência não foi encontrada.]**. This finite element requires a moderate number of degrees of freedom for accuracy and is not prone to shear locking. It has other known advantages of  $p$ -elements **[Erro! A origem da referência não foi encontrada.]**, like the fact the number of finite elements is determined by the geometry of the structure to study, rather than by precision requirements. Consequently, the user can define a precise computational model without much effort.

In this paper, the  $p$ -version element presented in **[Erro! A origem da referência não foi encontrada.]** is employed to study forced, *i.e.*, non-autonomous, oscillations of

symmetric laminated plates in the non-linear regime. The shear deformation and rotary inertia are considered in the model and the force in the direction transverse to the plate is sinusoidal in time. Direct numeric integration of the equations of motion is carried out by Newmark's method [**Erro! A origem da referência não foi encontrada.**]. The Poincaré maps, phase plots and time histories are shown, and the largest Lyapunov exponent of a chaotic oscillation is computed.