Time Domain Analysis of Geometrically Nonlinear Vibrations of Composite Laminated Plates By The Hierarchical Finite Element Method

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Composite laminated plates are employed in many areas of engineering such as aeronautics, space engineering and naval industry [1]-[3]. For example in aircrafts they are submitted to large acoustic, aerodynamic and inertia excitations and therefore experience vibrations with large amplitude, i.e., in the geometrically nonlinear regime. Vibrations with large amplitude cause large stresses and the diminution of life due to fatigue. Quasi periodic and chaotic behaviours are other possible consequences of non-linearity, completely ignored by the linear models which are normally used in engineering design.

Non-symmetrical laminated plates have been scarcely studied, particularly in non-linear dynamics. In this work, a hierarchical finite element, in which the model is improved by increasing the number of shape functions in each element, is formulated. The element is general, in the sense that it applies to symmetric or asymmetric plates. Compared with *h*-version elements, it has the major advantages of requiring a small number of degrees of freedom for accuracy and of being free from shear locking.

Different shear deformation theories (SDT) have been developed to include the effect of transverse shear deformation [1]-[3]. The first order shear deformation laminated plate theory is followed here. Hence, Kirchhoff's hypothesis [4] is relaxed by removing the third part, that is the transverse normals do not remain perpendicular to the mid-surface after deformation.

In this paper, the shape functions used are the Legendre polynomials in the Rodrigue's form [5]-[9]. A different number of transverse, middle plane, rotation about y and rotation about x, displacement shape functions are employed, and the convergence of the linear natural frequencies studied.

Two symmetric, rectangular, graphite/epoxy composite laminated plates with constant thickness *h*, width *a* and length *b*, composed of orthotropic layers oriented at different angles θ are considered. The element derived is employed to study the non-linear dynamic behaviour of some these laminated plates, in the time domain. The direct integration of the system of equations of motion is carried out by Newmark's method. In order to uncover the characteristics of the motions computed, time plots, phase planes and Fourier spectra are defined.

For different fibre orientations, it is confirmed that the linear natural frequencies of the thick plate model are smaller than the thin plate ones. Harmonic excitations are applied, and the responses of the plates are determined with a reasonable computational cost, due to the reduced number of degrees of freedom of the model. For amplitudes of vibration much lower than the thickness of the plate, the solutions found are always periodic and highly dominated by the harmonic with frequency equal to the excitation frequency (principal harmonic). Increasing the force applied, the response amplitude increased in a non-linear way, the motions found are still periodic, but the time and phase plots show that higher harmonics are important.

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